

# Revisiting constraints on (pseudo)conformal Universe with Planck data

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## Abstract

We revisit constraints on the (pseudo)conformal Universe from the non-observation of statistical anisotropy in the Planck data. The quadratic maximal likelihood estimator is applied to the Planck temperature maps at frequencies 143 GHz and 217 GHz as well as their cross-correlation. The strongest constraint is obtained in the scenario of the (pseudo)conformal Universe with a long intermediate evolution after conformal symmetry breaking. In terms of the relevant parameter (coupling constant), the limit is  $h^2 < 0.0013$  at 95% C.L. (using the cross-estimator). The analogous limit is much weaker in the scenario without the intermediate stage ( $h^2 \ln \frac{H_0}{\Lambda} < 0.52$ ) allowing the coupling constant to be of order one. In the latter case, the non-Gaussianity in the 4-point function appears to be a more promising signature.

Statistical isotropy (SI) is one of the vanilla predictions of the slow roll inflation. Deviations from this property – if observed in the cosmic microwave background (CMB) or large scale structure surveys – would imply a non-trivial extension of the standard cosmology. Several models of inflation with vector fields have been recently put forward [1] (see [2] for a review) predicting an anisotropic Universe with a detectable statistical anisotropy (SA). It was pointed out, however, that many of these models suffer from ghosts [3] or rely on strong tuning in the parameter space [4]. These problems are absent in some alternatives

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to inflation i.e., models of the (pseudo)conformal Universe [5, 6, 7]. The latter are the main focus of this *Letter*.

In the pseudo(conformal) Universe, the space-time geometry is effectively Minkowskian at the times preceding the hot Big Bang. The state of the early Universe in this picture is described in terms of conformal field theory. The conformal symmetry is assumed to be spontaneously broken down to the de Sitter subgroup. The zero-weight conformal field present in the Universe at these early times evolves in the symmetry breaking background and its perturbations acquire flat power spectrum [5, 6, 7]. These field perturbations get reprocessed into adiabatic perturbations at much later epoch. The source of non-trivial phenomenology in this setup is the interaction between zero-weight field perturbations and the Goldstone field associated with the symmetry breaking pattern [8, 10, 11, 12]. In particular, very long wavelength modes of the Goldstone field give rise to SA [8, 9, 12], while shorter ones lead to non-Gaussianity (NG) [10, 12].

Concrete realizations of the (pseudo)conformal Universe include conformal rolling scenario [5] and Galilean genesis [6]. In these two models the conformal group is spontaneously broken down to the de Sitter subgroup by the homogeneous time-dependent solution of the unit conformal weight field  $\rho$ . The form of the solution is fixed by the dilatation invariance, which remains unbroken after spontaneous symmetry breaking,

$$\rho = \frac{1}{h(t_* - t)} .$$

The constant  $h$  here is the most important parameter of the conformal rolling scenario and Galilean genesis, as it governs the non-trivial phenomenology, including SA;  $t_*$  is the constant of integration, which has the meaning of the end-of-roll time.

At the level of primordial curvature perturbations  $\zeta$ , SA implies directional dependence of the power spectrum,

$$\mathcal{P}_\zeta(\mathbf{k}) \propto \left( 1 + \sum_{LM} q_{LM}(k) Y_{LM}(\hat{\mathbf{k}}) \right) . \quad (1)$$

Here  $Y_{LM}(\hat{\mathbf{k}})$  are the spherical harmonics,  $\hat{\mathbf{k}}$  is the direction of the perturbation wavevector  $\mathbf{k}$  and  $q_{LM}(k)$  are the coefficients parametrizing SA.

In the (pseudo)conformal Universe scenario, there are two alternative predictions concerning SA. One of them is obtained if cosmological modes are superhorizon by the end of the roll (at times close to  $t_*$ ). In that case, the directional dependence is of the quadrupolar form [8, 12],

$$q_{2M} = \frac{H_0}{k} q'_{2M} + q''_{2M} . \quad (2)$$

Here  $q'_{2M}$ 's and  $q''_{2M}$ 's encode contributions to SA appearing in the linear and quadratic orders in the parameter  $h$ , respectively. Note that the leading order (LO) contribution is

characterized by the decreasing amplitude; the Hubble rate  $H_0$  plays the role of the ultraviolet cutoff for infrared modes of the Goldstone field feeding into SA. Coefficients  $q'_{2M}$  obey the Gaussian statistics with the zero mean and the following dispersion

$$\langle q'_{2M} q'^{*}_{2M'} \rangle = \frac{\pi h^2}{25} \delta_{MM'} .$$

The sub-leading order (SLO) contribution is of the axisymmetric form. Namely,

$$q''_{2M} = -\frac{4\pi v^2}{5} Y_{2M}^*(\hat{\mathbf{v}})$$

Here  $\mathbf{v}$  is the Gaussian vector related to the Goldstone field. Its components have zero means and dispersions

$$\langle v_i^2 \rangle = \frac{3h^2}{8\pi^2} \ln \frac{H_0}{\Lambda} ,$$

where  $\Lambda$  is the infrared cutoff for the modes of the Goldstone field;  $\hat{\mathbf{v}} = \mathbf{v}/v$  is a unit vector.

Another prediction is obtained if cosmological modes of interest are still subhorizon by the end of the roll. After the rolling stage, they proceed to evolve at the so called intermediate stage [9, 13]. The structure of SA in this case is particularly rich. Namely, all the coefficients  $q_{LM}$  with even  $L$  are non-zero in (1). They are the Gaussian quantities with zero means and variances given by [9]

$$\langle q_{LM} q_{L'M'}^* \rangle = \tilde{Q}_L h^2 \delta_{LL'} \delta_{MM'} , \quad (3)$$

where

$$\tilde{Q}_L = \frac{3}{\pi} \times \frac{1}{(L-1)(L+2)} .$$

In what follows, we use the notion “sub-scenario A” for the version of (pseudo)conformal Universe without the intermediate stage, Eq. (2), and “sub-scenario B” for the version with intermediate stage, Eq. (3).

In order to derive the coefficients  $q_{LM}$  from the CMB data, we make use of the quadratic maximal likelihood (QML) estimator first constructed in Ref. [14]. This has proved to be a powerful tool for studies of SA in WMAP [14, 15, 16] and Planck data [17]. Furthermore, QML methodology of the data analysis results in an excellent agreement with the exact methods used in Ref. [18]. The estimator is a quadratic form in the space of maps. In this paper we rewrite the estimator as a bilinear function of two maps in a way similar to the WMAP cross-power spectrum [19]. Given the spherical harmonic coefficients for the two maps  $\hat{a}_{lm}^i, \hat{a}_{lm}^j$ , where  $i, j$  denote the frequency band, we express both the original estimator  $i = j$  and the new cross-estimator  $i \neq j$  in the following way

$$q_{LM}^{ij} = \sum_{L'M'} (\mathbf{F}^{ij})_{LM;L'M'}^{-1} (h_{L'M'}^{ij} - \langle h_{L'M'}^{ij} \rangle) , \quad (4)$$

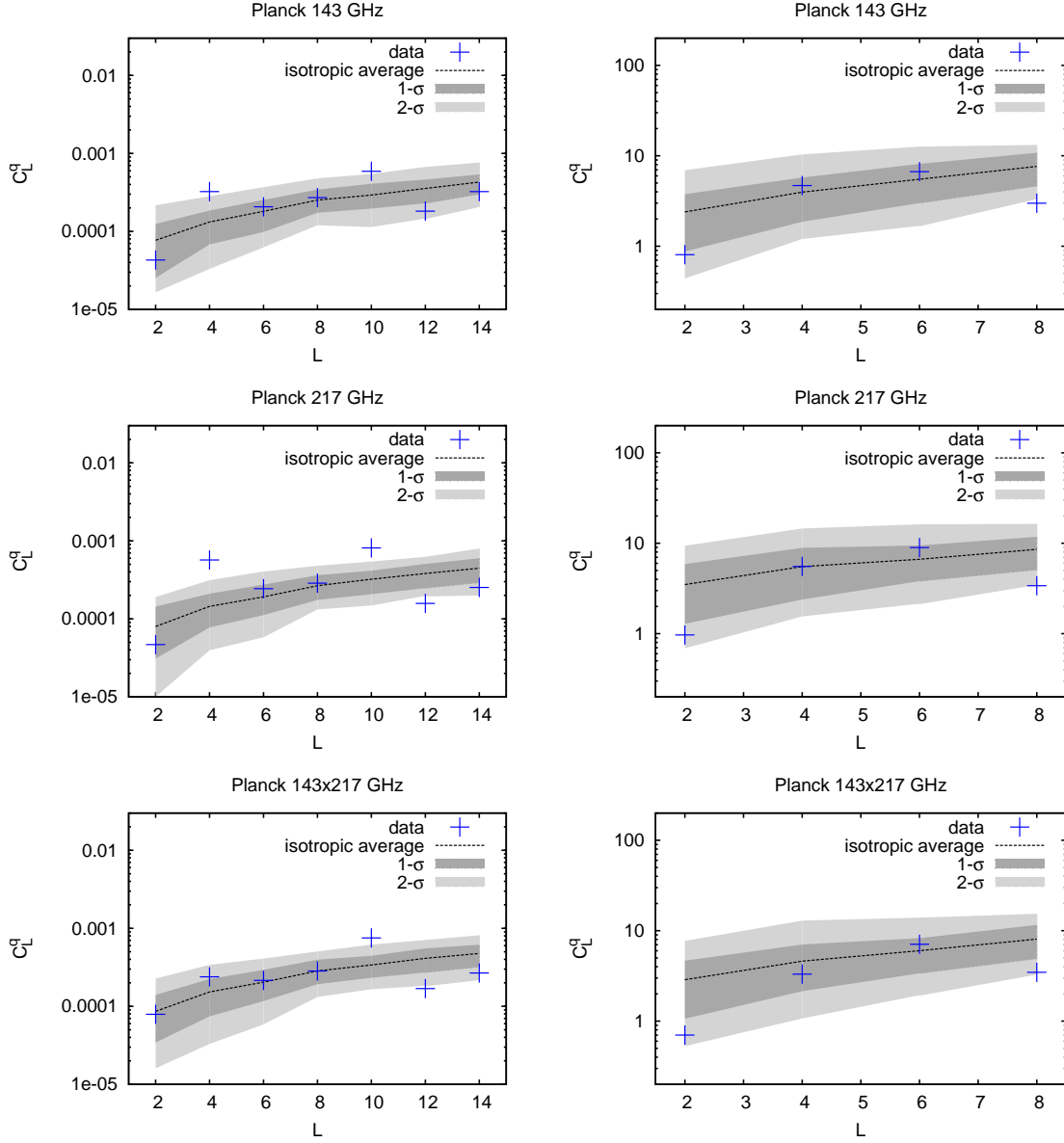


Figure 1: Coefficients  $C_L^q$  given by Eq. (9) reconstructed from the Planck data. Plots in the left and in the right column correspond to the choice  $a(k) = 1$  and  $a(k) = H_0 k^{-1}$  in Eq. (6), respectively. 68% and 95% C.L. intervals are overlaid with dark grey and light grey, respectively.

where  $\langle \rangle$  denotes the averaging over different realizations of isotropic maps and

$$h_{LM}^{ij} = \sum_{l'l'mm'} \frac{1}{2} i^{l'-l} C_{l'l'} B_{lm;l'm'}^{LM} \bar{a}_{l,-m}^i \bar{a}_{l'm'}^j ; \quad (5)$$

$\bar{a}_{lm}^i$  are the CMB temperature coefficients filtered with the inverse isotropic covariance,

$$\bar{a}_{lm} = (\mathbf{S}^{iso} + \mathbf{N}^i)_{lm;l'm'}^{-1} \hat{a}_{l'm'} .$$

Here  $\mathbf{S}^{iso}$  is a theoretical isotropic covariance, and  $\mathbf{N}$  is the noise matrix. The coefficients  $C_{ll'}$  in Eq. (5) are given by

$$C_{ll'} = 4\pi \int d\ln k \Delta_l(k) \Delta_{l'}(k) a(k) \mathcal{P}_\zeta(k) , \quad (6)$$

where  $\Delta_l(k)$  is a transfer function. The function  $a(k)$  here encodes the possible dependence of SA on the scale  $k$ . It is given by  $a(k) = H_0 k^{-1}$  for the LO contribution in sub-scenario A; it should be set to unity in other cases. Coefficients  $B_{lm;l'm'}^{LM}$  are expressed in terms of the Wigner 3j-symbols,

$$B_{lm;l'm'}^{LM} = (-1)^M \sqrt{\frac{(2L+1)(2l+1)(2l'+1)}{4\pi}} \begin{pmatrix} L & l & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & l & l' \\ M & m & -m' \end{pmatrix} .$$

Finally,  $\mathbf{F}^{ij}$  in Eq. (4) is the Fisher matrix defined by

$$F_{LM;L'M'}^{ij} \equiv \langle h_{LM}^{ij} (h_{L'M'}^{ij})^* \rangle - \langle h_{LM}^{ij} \rangle \langle (h_{L'M'}^{ij})^* \rangle . \quad (7)$$

The analytic expression for the Fisher matrix in the homogeneous noise approximation is given by

$$F_{LM;L'M'}^{ij} = \delta_{LL'} \delta_{MM'} f_{\text{sky}} \sum_{l,l'} \frac{(2l+1)(2l'+1)}{16\pi} \begin{pmatrix} L & l & l' \\ 0 & 0 & 0 \end{pmatrix}^2 \frac{C_{ll'}^2 \cdot (C_l^{\text{tot},i} C_{l'}^{\text{tot},j} + \tilde{C}_l^i \tilde{C}_{l'}^j)}{(C_l^{\text{tot},i})^2 (C_{l'}^{\text{tot},j})^2} . \quad (8)$$

Here  $C_l^{\text{tot},i} = C_l + N_l^i$ , where  $C_l$  is the standard CMB angular power spectrum and  $N_l^i$  is the angular power spectrum of the homogeneous noise;  $\tilde{C}_l^i = C_l^{\text{tot},i}$  for  $i = j$  and  $\tilde{C}_l^i = C_l$  in the opposite case. We hide the details of the derivation of this formula for the case of the single-frequency band analysis in Appendix.

Model/band	143 GHz	217 GHz	143 × 217 GHz
Sub-scenario A (LO)	$h^2 < 8.8$	$h^2 < 8.0$	$h^2 < 3.0$
Sub-scenario A (NLO)	$h^2 \ln \frac{H_0}{\Lambda} < 0.34$	$h^2 \ln \frac{H_0}{\Lambda} < 0.30$	$h^2 \ln \frac{H_0}{\Lambda} < 0.52$
Sub-scenario B	$h^2 < 0.0011$	$h^2 < 0.0090$	$h^2 < 0.0013$
Inflation	$ g_*  < 0.020$	$ g_*  < 0.020$	$ g_*  < 0.026$

Table 1: Planck 95% C.L. constraints on the parameter  $h^2$  of (pseudo)conformal Universe and on the amplitude of the axisymmetric quadrupole anisotropy  $g_*$ .

We use the Planck CMB temperature maps corresponding to the first 15.5 months of observation at the frequencies 143 GHz and 217 GHz [20, 21]. To remove the contamination of the galactic light and point sources we apply the High Frequency Instrument (HFI) power spectrum mask [21, 22], which leaves 43% of the sky unmasked. Averaging over statistically isotropic realizations is performed using 100 Planck simulated multi-frequency CMB maps coadded with the corresponding noise maps and the foreground maps [21, 23]. These maps incorporate the effects of beam asymmetries and complex scanning strategy [24], which are proved to be crucial for the SI studies [25, 23, 17]. The operations with the maps are performed with the *HEALPix* and *healpy* packages [26].

We implement the estimator (4) in several steps. First, we carry out the inverse-variance filtering using the multigrid preconditioner [27]; the procedure is discussed in detail in Ref. [14]. Second, we evaluate the coefficients  $C_{\ell}$  using *CAMB* [28] and, finally, calculate the sum in Eq. (5) using *gsl* [29] and *slatec* [30] libraries. The range of the multipoles studied is set to  $2 \leq l \leq 1600$ . For the larger values of  $l$ , the signal is dominated by the instrumental noise.

In Fig. 1 we plot the coefficients

$$C_L^q = \frac{1}{2L+1} \sum_M |q_{LM}|^2 \quad (9)$$

estimated from the data at frequencies 143 GHz and 217 GHz and their cross-correlation. As it is clearly seen, at the level of the quadrupole the Planck data are in agreement with the hypothesis of SI. On the other hand, the power of anisotropy contained in  $L = 4$  of 143 and 217 GHz frequency bands deviates from SI expectations at more than  $2\sigma$  level. We note that the amplitude and the orientation of the  $L = 4$  multipole are different for different frequency bands. Moreover, the significance of the excess grows with the increase of the cutoff  $l_{max}$ . Finally, the signal is consistent with the hypothesis of SI in the cross-estimator. Therefore we argue that the enhancement at  $L = 4$  may originate from statistical fluctuation or systematic effects of the noise. The noise dominates at high multipoles and is completely uncorrelated between the bands. Consequently the cross-correlation effectively wipes out all the noise-related effects. In what follows we consider the cross-estimator as the preferred method for the study of SA with the Planck data.

We start with constraining the models of the (pseudo)conformal Universe, where the intermediate stage is absent (sub-scenario A). To estimate the parameter  $h^2$ , we use the statistics given by the coefficient  $C_2^q$ . The remainder of the procedure parallels that employed in Ref. [16]. The final constraints originating from the LO and SLO contributions to SA are presented in Table 1. We note that the limit from the SLO term in Eq. (2) is in fact stronger. This is not a surprise, as the LO term decreases with the wavenumber  $k$ . Hence, it leaves a weaker imprint on CMB for the relevant values of the parameter  $h$ , in agreement with the findings of previous works [15, 16].

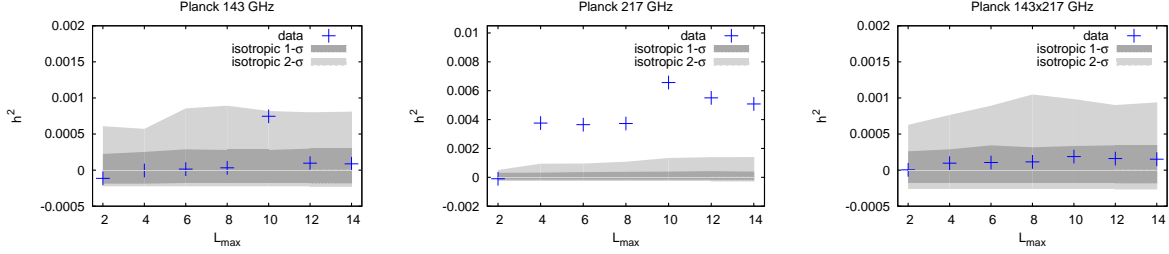


Figure 2: Parameter  $h^2$  of the (pseudo)conformal Universe with long intermediate stage estimated from the Planck data. 68% and 95% C.L. intervals are overlaid with dark grey and light grey, respectively.

In Table 1 we also present the constraint on the amplitude  $g_*$  of the SA of the axisymmetric type,  $\mathcal{P}_\zeta(\mathbf{k}) \propto (1 + g_* \cos^2 \theta)$ , where  $\theta$  is the angle between the perturbation wavevector  $\mathbf{k}$  and the direction of SI breaking. We do this in view of several inflationary scenarios in which SA of this type is generated by vector fields [1]. Our limits demonstrate a relatively mild improvement as compared to ones of Ref. [17], which may be attributed to the use of the inverse-variance filtering. We leave more detailed study of the anisotropic inflationary models for our future work [31].

To constrain the parameter  $h^2$  in the versions of the (pseudo)conformal Universe with a long intermediate stage (sub-scenario B), we use the estimator [15]

$$h^2 \sum_L \frac{(2L+1)F_L^2 \tilde{Q}_L^2}{(1 + F_L \tilde{Q}_L h^2)^2} = \sum_L \frac{(2L+1)F_L \tilde{Q}_L}{(1 + F_L \tilde{Q}_L h^2)^2} (F_L C_L^q - 1), \quad (10)$$

Here  $F_L$  are the elements of the Fisher matrix, which we assume to be diagonal. The results of implementing this estimator to the Planck data are shown in Fig. 2. Values of the estimators are plotted for seven ranges of the multipoles starting from the quadrupole  $L = 2$  and extending up to  $L_{max} = 2, \dots, 14$ . The results for the frequency band 217 GHz clearly exhibit SA, which is again due to the enhancement at  $L = 4$ , see Fig. 1. The rest of the procedure is the same as in the previous papers [15, 16]. Final constraints are presented in Table 1.

We conclude that statistical anisotropy is a significant signature in the sub-scenario B of the (pseudo)conformal Universe, while it is relatively weak in the sub-scenario A. Fortunately, the latter may yield strong NG at the trispectrum level [10, 12]. The leading contribution to the NG in these models occurs in the order  $h^2$ . Given its mild behavior in the folded limit, i.e., when two cosmological momenta are anticollinear, it can be compared with the equilateral type NG (rather conservatively). Using the existing WMAP limit on the corresponding trispectrum parameter  $|\tau_{NL}^{equil}| \lesssim 7 \times 10^6$  [32] and the estimates presented in [8], one expects

to arrive at the constraint  $h^2 \lesssim 0.1 - 1$ . This is already comparable with the constraint deduced from the non-observation of SA. There is in fact another contribution to the NG [12]. Though it emerges in the quartic order in the constant  $h$ , this contribution is enhanced in the folded limit relative to the LO one. Remarkably, the SLO NG is precisely of the local type, at least in the folded limit. Making use of the Planck 95% C.L. limit  $|\tau_{NL}^{loc}| < 2800$  [33], one would expect the constraint as strong as  $h^2 \lesssim 0.01 - 0.1$ . Hence, NG appears to be the most promising signature of the (pseudo)conformal Universe without the intermediate stage. The detailed analysis of the trispectrum remains to be performed, however.

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## Appendix

In this Appendix, we provide the analytical computation of the Fisher matrix in the homogeneous noise approximation for the realistic case of the masked sky. We do this for the single-frequency band analysis. The generalization to the multi-frequency-band analysis is straightforward. We start with substituting Eq. (5) into Eq. (7) and obtain

$$F_{LM;L'M'} = (-1)^{M'} \frac{1}{4} \sum_{l'l';mm'} \sum_{\tilde{l}'\tilde{m}'\tilde{m}'} i^{l'-l+\tilde{l}-\tilde{l}'} C_{ll'} C_{\tilde{l}\tilde{l}'} B_{lm;l'm'}^{LM} B_{\tilde{l}\tilde{m};\tilde{m}'\tilde{m}'}^{L',-M'} \left( \langle \bar{a}_{l,-m} \bar{a}_{\tilde{l},-\tilde{m}} \rangle \langle \bar{a}_{l'm'} \bar{a}_{\tilde{l}',\tilde{m}'} \rangle + \langle \bar{a}_{l,-m} \bar{a}_{\tilde{l}',\tilde{m}'} \rangle \langle \bar{a}_{\tilde{l},-\tilde{m}} \bar{a}_{l'm'} \rangle \right). \quad (11)$$

Here we made use of the Isserlis-Wick theorem. The relation between the coefficients  $\hat{a}_{lm}$  and spectral coefficients obtained from the hypothetical full sky analysis  $\hat{a}_{lm}^f$  is given by [34]

$$\hat{a}_{lm} = \sum_{l'm'} W_{lm;l'm'} \hat{a}_{l'm'}^f.$$

Here  $W_{lm;l'm'}$  is the transition matrix

$$W_{lm;l'm'} = \int d\mathbf{n} W(\mathbf{n}) Y_{lm}^*(\mathbf{n}) Y_{l'm'}(\mathbf{n}). \quad (12)$$

$W(\mathbf{n})$  is the “mask” function, which takes in the case of the sharp mask the value 1 in the unmasked pixels and zero otherwise. Then the unmasked fraction of the sky  $f_{sky}$  is given by the integral of  $W(\mathbf{n})$  over the sphere,

$$f_{sky} = \int \frac{d\mathbf{n}}{4\pi} \cdot W(\mathbf{n}). \quad (13)$$



Neglecting the commutator between the masking and inverse-variance filtering procedures, one arrives to the following relation for the filtered harmonic coefficients

$$\bar{a}_{lm} = \sum_{l'm'} W_{lm;l'm'} \bar{a}_{l'm'}^f . \quad (14)$$

Substituting Eq. (14) into Eq. (11), we obtain

$$\begin{aligned} F_{LM;L'M'} &= (-1)^{M'} \frac{1}{2} \sum_{ll';mm'} \sum_{\tilde{l}\tilde{l}';\tilde{m}\tilde{m}'} i^{l'-l+\tilde{l}-\tilde{l}'} C_{ll'} C_{\tilde{l}\tilde{l}'} \left( C_l^{tot} C_{\tilde{l}}^{tot} C_{l'}^{tot} C_{\tilde{l}'}^{tot} \right)^{-1} B_{lm;l'm'}^{LM} B_{\tilde{l}\tilde{m};\tilde{l}'\tilde{m}'}^{L',-M'} \times \\ &\times \sum_{n=0}^{\infty} \sum_{k=-n}^n \sum_{n'=0}^{\infty} \sum_{k'=-n'}^{n'} (-1)^{k+k'} C_n^{tot} C_{n'}^{tot} W_{ln;-m,k} W_{\tilde{l}n;-\tilde{m},-k} W_{l'n';m',k'} W_{\tilde{l}'n';\tilde{m}',-k'} . \end{aligned} \quad (15)$$

Here we made use of the following relations, which are valid in the homogeneous noise approximation

$$\bar{a}_{lm}^f = (C_l^{tot})^{-1} \hat{a}_{lm}^f ,$$

and

$$\langle \hat{a}_{lm}^f \hat{a}_{l'm'}^f \rangle = (-1)^m C_l^{tot} \delta_{ll'} \delta_{m,-m'} .$$

Replacing sufficiently slowly varying functions  $C_n^{tot}$  and  $C_{n'}^{tot}$  by  $C_l^{tot}$  and  $C_{l'}^{tot}$ , we obtain

$$\begin{aligned} F_{LM;L'M'} &\approx \frac{1}{2} \sum_{ll';mm'} \sum_{\tilde{l}\tilde{l}';\tilde{m}\tilde{m}'} i^{l'-l+\tilde{l}-\tilde{l}'} (-1)^{M+m+\tilde{m}+m'+\tilde{m}'} C_{ll'} C_{\tilde{l}\tilde{l}'} \left( C_l^{tot} C_{\tilde{l}}^{tot} \right)^{-1} \times \\ &\times \int d\mathbf{n} Y_{lm}(\mathbf{n}) Y_{l',-m'}(\mathbf{n}) Y_{LM}(\mathbf{n}) \int d\tilde{\mathbf{n}} Y_{\tilde{l}\tilde{m}}(\tilde{\mathbf{n}}) Y_{\tilde{l}',-\tilde{m}'}(\tilde{\mathbf{n}}) Y_{L',-M'}(\tilde{\mathbf{n}}) \times \\ &\times \int d\mathbf{n}_1 W(\mathbf{n}_1) Y_{lm}(\mathbf{n}_1) Y_{\tilde{l}\tilde{m}}(\mathbf{n}_1) \int d\mathbf{n}_2 W(\mathbf{n}_2) Y_{l',-m'}(\mathbf{n}_2) Y_{\tilde{l}',-\tilde{m}'}(\mathbf{n}_2) . \end{aligned} \quad (16)$$

Let us comment on the derivation of this formula. First, we replaced the coefficients  $W_{lm;l'm'}$  in Eq. (15) by Eq. (12) and coefficients  $B_{lm;l'm'}^{LM}$  by the integral over spherical harmonics

$$B_{lm;l'm'}^{LM} = \int d\mathbf{n} Y_{lm}^*(\mathbf{n}) Y_{l'm'}(\mathbf{n}) Y_{LM}(\mathbf{n}) ,$$

respectively. We then provided the summation over the indexes  $n$ ,  $k$ ,  $n'$  and  $k'$ . At this point the following relation is used

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l Y_{lm}^*(\mathbf{n}) Y_{lm}(\mathbf{n}') = \delta(\mathbf{n} - \mathbf{n}') \quad (17)$$

The expression (16) is obtained by integrating out the delta-functions. Next, we again replace slowly changing functions  $C_{\tilde{l}}$  and  $C_{\tilde{l}'}$  by  $C_l$  and  $C_{l'}$ , respectively and sum over  $(\tilde{l}, \tilde{m})$  and

$(\tilde{l}', \tilde{m}')$  in Eq. (16) using Eq. (17). We arrive at

$$F_{LM;L'M'} \approx \frac{1}{2} \sum_{ll';mm'} \frac{C_{ll'}^2}{C_l^{tot} C_{l'}^{tot}} \int d\mathbf{n} Y_{l,-m}(\mathbf{n}) Y_{l'm'}(\mathbf{n}) Y_{L,-M}^*(\mathbf{n}) \times \\ \times \int d\mathbf{n}' W(\mathbf{n}') Y_{l,-m}^*(\mathbf{n}') Y_{l'm'}^*(\mathbf{n}') Y_{L',-M'}(\mathbf{n}') . \quad (18)$$

Finally, we sum over  $m$  and  $m'$  indexes,

$$F_{LM;L'M'} \approx \frac{1}{2} \sum_{ll'} \left( \frac{2l+1}{4\pi} \right) \left( \frac{2l'+1}{4\pi} \right) \frac{C_{ll'}^2}{C_l^{tot} C_{l'}^{tot}} \times \\ \times \int d\mathbf{n} d\mathbf{n}' W(\mathbf{n}') P_l(\mathbf{n}\mathbf{n}') P_{l'}(\mathbf{n}\mathbf{n}') Y_{L,-M}^*(\mathbf{n}) Y_{L',-M'}(\mathbf{n}') . \quad (19)$$

The expression (19) may be simplified by using the fact that the matrix (19) is approximately diagonal, and has a sufficiently mild dependence on the numbers  $M$  and  $M'$ . The Fisher matrix may be therefore approximated as

$$F_{LM;L'M'} \approx F_L \delta_{LL'} \delta_{MM'} ,$$

where

$$F_L \approx \frac{1}{2L+1} \sum_M F_{LM;LM} . \quad (20)$$

Taking the sum over  $M$  index we arrive at

$$F_L \approx \frac{1}{8\pi} \sum_{ll'} \left( \frac{2l+1}{4\pi} \right) \left( \frac{2l'+1}{4\pi} \right) \frac{C_{ll'}^2}{C_l^{tot} C_{l'}^{tot}} \int d\mathbf{n} d\mathbf{n}' W(\mathbf{n}') P_l(\mathbf{n}\mathbf{n}') P_{l'}(\mathbf{n}\mathbf{n}') P_L(\mathbf{n}\mathbf{n}') .$$

The integral over  $\mathbf{n}$  may be taken,

$$F_L \approx \sum_{ll'} \frac{(2l+1)(2l'+1)}{8\pi} \frac{C_{ll'}^2}{C_l^{tot} C_{l'}^{tot}} \left( \begin{matrix} L & l & l' \\ 0 & 0 & 0 \end{matrix} \right)^2 \int \frac{d\mathbf{n}'}{4\pi} \cdot W(\mathbf{n}')$$

Taking into account Eq. (13), we arrive at Eq. (8) of the main text of the paper. In particular, the calculation justifies the presence of the factor  $f_{sky}$  in the approximate Fisher matrix.

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